

Six Dimensional Schwarzschild Black Holes in M(atrix) Theory

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We calculate the entropy of six dimensional Schwarzschild black holes in matrix theory. We use the description of the matrix model on T^5 as the world-volume theory of NS five-branes and show that the black hole entropy is reproduced by noncritical closed strings with fractional tension living on the five-brane.

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In recent years there has been great progress in understanding the microscopic structure of extreme and near-extreme black holes; however the same cannot be said for Schwarzschild black holes. The reason for this is the fact that Schwarzschild black holes are not even nearly supersymmetric systems. Thus the powerful methods of supersymmetry and D brane physics are not applicable in this case. Nevertheless, there has been some understanding of Schwarzschild black holes in string theory[1,2,3]. In ref. [2] it was shown that the black hole entropy in any dimension can be obtained if one assumes that they are described by highly excited fundamental strings and takes the gravitational redshift of the string mass into account. The microscopic description of black holes requires knowledge of strongly coupled string theory since these are not perturbative systems. At the moment the only candidate for nonperturbative string theory (M theory) is the matrix model[4]. Thus it is important to understand black holes in the matrix model and see if this enhances our understanding of the problem beyond string theory. Extreme and near-extreme matrix black holes in five dimensions have already been considered in [5,6,7]. Schwarzschild black holes in matrix theory in eight and other dimensions also have been recently investigated in [8,9].

In this letter, we consider six dimensional Schwarzschild black holes in the matrix model in the Discrete Light-Cone Quantization (DLCQ) formulation[10]. Since the matrix model is formulated in a highly boosted frame with a large light-cone momentum, the original Schwarzschild black hole becomes a near-extreme system. We describe the matrix model compactified on T^5 as the world-volume theory of N NS five-branes (of IIA string theory) with a transverse compact circle[11]. The five brane world-volume theory contains strings whose tension is fixed by the size of the transverse circle (since these are membranes which wrap around the circle and end on the five-brane). The D=6 Schwarzschild black hole is now described by the deviation from extremality of N NS five-branes. The original longitudinal momentum modes (partons) in space-time become NS five-branes and the Schwarzschild black hole is described by non-BPS excitations of this system.

On the other hand, it is known that the entropy of a near-extreme five-branes is given by a noncritical, non-BPS closed string which lives on the 5+1 dimensional world-volume[13,12]. In order to get the proper thermodynamic description of the black hole one assumes that the tension of the string is fractional. In the following we will assume that the strings which give the five-brane entropy are the same ones that appear as membranes which end on the NS five-branes. We will see that this reproduces the correct black hole entropy up to

numerical factors and shows how the string tension becomes fractional. One problem with this approach is the fact that the strings must be weakly interacting for entropy to workout correctly. However, it is known that the noncritical strings living on the five-branes are self-dual and therefore strongly coupled.

We consider the matrix model in the DLCQ formulation, i.e. for finite light-cone momentum $P = N/R$ where N and the light-cone radius R are finite. The lowest possible value of N needed to count the black hole entropy is given by an argument from ref. [8]. Consider a black hole with radius R_s in its rest frame which is also extended along the light-cone direction. Now it is possible that $R_s > R$ and the black hole will not fit in the light-cone direction. However, one can boost the black hole along the longitudinal direction and then its longitudinal size contracts to

$$R' \sim \frac{M}{P} R_s = \frac{MR}{N} R_s \quad (1)$$

We see that in order for the black hole to fit into longitudinal size R the boost must be at least

$$N > MR_s \quad (2)$$

For a Schwarzschild black hole in D space-time dimensions the radius is given by ($G_D = \ell_{11}^9/L^{11-D}$ is the D dimensional Newton constant.)

$$R_s \sim (G_D M)^{1/D-3} \quad (3)$$

so that the boost parameter becomes

$$N \sim G_D^{1/D-3} M^{D-2/D-3} \quad (4)$$

which is the Schwarzschild black hole entropy in D dimensions! In the following we will use this value of N in our calculations. One can interpret this as the minimal value of N that can result in the resolution needed to count the black hole entropy. Note that with this amount of boost the original Schwarzschild black hole (which was a highly nonextreme system) becomes near-extreme. The system now looks like N partons of the matrix model which are slightly excited.

We consider $D = 6$ Schwarzschild black holes which means that we compactify the matrix model on T^5 . For simplicity we compactify on T^5 with all sides equal to L . This compactification of the matrix model is described by the world-volume theory of N coincident NS five-branes (of IIA string theory) with a compact transverse direction. The size of the five-brane world-volume is given by

$$\Sigma_{1,2,3,4} = \frac{\ell_{11}^3}{RL} \quad \Sigma_5 = \frac{\ell_{11}^6}{RL^4} \quad (5)$$

and the radius of the transverse circle is

$$M_s^2 = \frac{R^2 L^5}{\ell_{11}^9} \quad (6)$$

The 5+1 dimensional world-volume theory has (2,0) supersymmetry. The sixteen BPS states of the matrix model on T^5 can be realized as zero, two and four branes inside the NS five branes. In addition, the longitudinal membranes and five-branes are given by momentum modes and wound BPS strings on the five-brane. All these states are bound to the five-branes since the IIA string coupling is vanishing. The moduli space is given by

$$\mathcal{M} = \frac{(R^4 \times S^1)^N}{S^N} \quad (7)$$

and the system is at the singular point of the moduli space at which all the five-branes coincide.

The Schwarzschild black hole is a non-BPS excitation of the N NS five-branes. It is known that the entropy of near-extreme five-branes is given by that of noncritical closed strings living on the five-brane world-volume. These “instantonic strings” satisfy

$$\frac{c_{eff}}{6} \frac{T}{T_{eff}} = N \quad (8)$$

In order to get the correct thermodynamics for low energy states of the black hole one assumes that the “instantonic strings fractionate”, i.e $c_{eff} = 6$ and $T_{eff} = T/N$. The Hagedorn temperature of these strings with fractional tension gives the Hawking temperature of the black hole. In the T^5 compactification of the matrix model there are also strings which

are membranes wrapped around the transverse circle ending on the five-brane. Their tension is M_s^2 . These are the strings which will give the black hole entropy. The string tension fractionates if we assume that in the black hole regime the NS five-branes are equally separated from each other rather than being on top of each other. This configuration corresponds to a nonsingular point in the moduli space. Then, the distance between consecutive five-branes becomes M_s^2/N and therefore the tension of the strings become fractional. This kind of tension fractionation also plays a role in extreme and near-extreme black holes[7].

We can now calculate the entropy of the $D = 6$ Schwarzschild black hole. From eq. (4) we find that

$$N \sim M^{4/3} G_6^{1/3} \sim S \quad (9)$$

For the noncritical strings living in six dimensions

$$E^2 = \frac{n}{N} M_s^2 \quad (10)$$

where n is the oscillator level and the factor N appears due to fractionation of the string tension. The entropy of the string is

$$S = 2\pi \sqrt{\frac{c}{6} n} \quad (11)$$

The world-volume energy of the string corresponds to the light-cone energy in space-time which means

$$E = \frac{R}{N} M^2 \quad (12)$$

Using eqs. (9), (10) and (12) the entropy becomes

$$S \sim N G_6^{1/2} R M_s^{-1} \quad (13)$$

Noting that the six dimensional Newton constant is $G_6 = L^5/\ell_{11}^9$ and using eq. (6) for M_s we find

$$S \sim N \sim M^{4/3} G_6^{1/3} \quad (14)$$

which is the correct six dimensional black hole entropy up to numerical factors. We see that the black hole entropy is reproduced correctly by a string living on the five-branes. This

string is the boundary of a membrane stretched between two five-branes a distance M_s^2/N apart and therefore has fractional tension. In order to use the string entropy formula, eq. (11) we assumed that these strings are weakly interacting. However, since these strings are self-dual their coupling is necessarily one. We do not know an explanation for this fact.

The six dimensional black hole was also considered in ref. [9] where the entropy was obtained by taking N coincident near-extreme D five-branes. Since the world-volume theory is now a 5+1 dimensional $U(N)$ SYM theory[16] the entropy will be given by a gas of particles and not strings[15]. Then the equation of state for a gas of particles in six dimensions is given by

$$S \sim \sqrt{N} E g_6 \quad (15)$$

where E is given by eq. (12) and the gauge coupling constant is

$$g_6^2 = \frac{\ell_{11}^9}{R^2 L^5} = M_s^{-1} \quad (16)$$

Using the above and eq. (9) we find that this equation of state is the same as that in eq. (13) arising from strings. The two descriptions of the six dimensional black hole are related by U duality. In fact in addition to the description of the matrix model on T^5 by NS five-branes of IIA string theory there is another T dual description in terms of NS five-branes of IIB string theory (which are S dual to D five-branes). The world-volume theory in this case is a $U(N)$ SYM theory with $(1,1)$ supersymmetry. However, this description is only correct at low world-volume energies. It still reproduces the correct black hole entropy since entropy is an ultraviolet effect in space-time and therefore it is related to infrared physics on the brane world-volume.

Acknowledgements

We would like to thank Tom Banks, Eva Silverstein and Lenny Susskind for discussions.

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